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# Bioconvection in a couple-stress fluid flow between concentric cylinders

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**--------------------ABSTRACT--------------------**
*The bioconvection in an annular region between concentric cylinders filled with couple stress fluid containing microorganisms is considered. The outer cylinder is assumed to be rotating while the inner cylinder is stationary. The governing equations are linearized using successive linearization method and then solved using Chebyshev collocation method. The influence of couple-stress parameter, bioconvection Peclet-number, Prandtl number, Lewis number and chemical reaction parameter on motile microorganism distribution are analyzed graphically. Also, the density number of motile microorganism is examined for various governing parameters along with slip parameter of concentration and motile microorganism*
***KEYWORDS;-*** *Bioconvection, Couplestress fluid, density number of motile microorganism*

## I. INTRODUCTION

The analysis of heat as well as mass transfer along with the fluid flow between concentric cylinders, with the outer/inner (one or both) cylinder(s) rotating/at stationary has significant applications in industry and engineering such as rotating machinery, drilling, swirl nozzles, heat exchangers, ballistics of projectiles with spin, chemical and mechanical mixing equipments, cooling systems in electronic devices, electrical machineries with rotor and stator etc.. Couette [1] was the first to initiate the study of viscous fluids flow in the annulus region between co-rotating cylinders, since then a wide range of theoretical investigations have been reported in the literature. Ahmed and Dutta [2] presented an analytical study of unsteady viscous incompressible fluid flow between two infinite concentric vertical cylinders with time dependent periodic pressure gradient subjected to applied magnetic field and in the presence of thermal radiation. Jha and Oni [3] analytically investigated the role of velocity slip and temperature jump on natural convection flow in a vertical micro-annulus. Roy et al [4] studied the natural convection flow in an annulus between a square enclosure and a circular cylinder filled with a chemically reacting fluid.

Most fluids used in engineering industrial applications such as suspensions of particles and fibers, slurries, polymer solutions and melts, surfactant, paints, foodstuffs, organic materials, adhesives, etc., do not satisfy the assumptions of a Newtonian fluid. In the literature, the behavior of such fluids is described by several models. Among these, couple stress fluid model introduced by Stokes [5] have specific features, such as non-symmetry of stress tensor, the presence of couple stresses and body couples. The key characteristic of couple stresses is to bring in a size dependent effect. The classical continuum mechanics fails to perceive this size effect of material particles within the continua. Several researchers have studied the flow of couple stress fluids in different geometries with various effects. Srinivasacharya and Kaladhar [6] investigated the hall and ion-slip effects on the mixed convective flow of couple stress fluid between two circular cylinders. Devakar et al [7] obtained the analytical solutions for the couple-stress fluid flow between two concentric cylinders using the slip boundary conditions. Nagaraju et al [8] attempted to analyze the influence of a radial magnetic field on the entropy generation in a couple stress fluid flow between vertical concentric cylinders, which are rotating, with a porous lining attached to the inside of an outer cylinder.

Bioconvection occurs in dense cultures of free swimming micro-organisms in appropriate aqueous environments, such as oceans and rivers, puddles, and droplets. Bioconvection has many applications in biological systems and biotechnology. Childress et al. [9] developed the first theory of gravitactic bioconvection for geotactic microorganisms. A continuum model for bioconvection in a suspension of swimming microorganisms was developed by Pedley [10]. Raees et al. [11] considered the unsteady mixed nano-bioconvection flow in a horizontal channel. Mosayebidorcheh et al. [12] analyzed the bioconvection flow in a horizontal channel filled with nanofluid which contains both nanoparticles and gyrotactic microorganisms. Zhao et al [13] studied the unsteady bioconvection squeezing flow in a horizontal channel with chemical reaction and magnetic field effects.

In this study, we investigate the bioconvection flow of couple stress fluid between concentric cylinders in which the outer cylinder is rotating. The influence of all non-dimensional physical parameters embedded in the bioconvection flow model is studied graphically on density of motile microorganism and density number of motile microorganism.

## II. MATHEMATICAL FORMULATION

A Consider the steady flow of an incompressible couple stress fluid containing gravitactic microorganisms in annuals between two concentric cylinders. A cylindrical polar coordinate system $(r, \varphi, z)$ is taken with $z$-axis as the common axis for both cylinders, as depicted in the Figure 1. The inner cylinder is at rest and the outer cylinder is rotating with constant angular velocity $\Omega$. The flow being generated due to the rotation of the outer cylinder. Let the radii of the inner and outer cylinders are $a$ and $b$, respectively. The inner and outer cylinders are maintained at the constant temperatures $T_a$ and $T_b$ , constant concentrations $C_a$ and $C_b$ and constant concentration of motile microorganisms $\vartheta_a$ and $\vartheta_b$ . With the above approximations, the equations governing present flow are given by

$$\frac{\partial u}{\partial \varphi} = 0 \tag{1}$$

$$\frac{\partial P}{\partial r} = \frac{\rho u^2}{r} \tag{2}$$

$$\eta_1 \nabla_1^4 u - \mu \nabla_1^2 u + \frac{1}{\rho}\frac{\partial \rho}{\partial \varphi} = 0 \tag{3}$$

$$\alpha^*\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \mu\left(\frac{\partial u}{\partial r} - \frac{u}{r}\right)^2 + \eta_1\left(\nabla_1^2 u\right)^2 = 0 \tag{4}$$

$$D_B\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}\right) - k_1(C - C_a) = 0 \tag{5}$$

$$\frac{b_c w_c}{C_1 - C_2}\frac{\partial}{\partial r}\left(\vartheta \frac{\partial C}{\partial r}\right) = D_n\left(\frac{\partial^2 \vartheta}{\partial r^2} + \frac{1}{r}\frac{\partial \vartheta}{\partial r}\right) \tag{6}$$

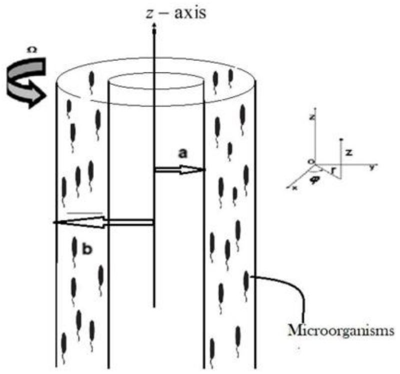



Figure 1: Physical model and coordinate system

with the following boundary conditions :

$$\left.\begin{array}{l} u = 0, \quad T = T_a, \quad C = C_a, \quad \vartheta = \vartheta_a \quad at \quad r = a \\ u = b\Omega, \quad T = T_b, \quad C = C_b, \quad \vartheta = \vartheta_b \quad at \quad r = b \end{array}\right\} \tag{7}$$

where u is the velocity components along the $\varphi$ direction, T is the temperature, C is the concentration, $\vartheta$ is the density of motile microorganism, $\mu$ is the fluid viscosity, $\rho$ is the density, $\alpha$ is the fluid thermal diffusivity, $D_b$ is the mass diffusivity, $D_n$ is the micro-organism diffusivity constant, $b_c$ is the Chemotaxis constant and $w_c$ is the maximum cell swimming speed, $\rho$ is the fluid density and

$$\nabla_1^2 = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru)\right)$$

Substituting the following Similarity transformation variables

$$r^2 = b^2\eta, u = \left(\frac{\Omega}{\sqrt{\eta}}\right) f(\eta),$$

$$T = T_0 + (T_a - T_0)\theta(\eta), C = C_0 + (C_a - C_0)\phi(\eta), \vartheta = \vartheta_a(\eta)$$

in equation (3) - (6), we get

$$S^2\eta^2 f'''' + 2S^2\eta f''' - \frac{1}{4}\eta f''^2 = 0 \quad (9)$$

$$\eta^3\theta'' + \eta^2\theta' + Br(\eta^2 f'^2 + f^2 - 2ff') + 4BrS^2\eta^3 f''^2 = 0 \quad (10)$$

$$\frac{1}{\Pr.Le}\eta\phi'' + \frac{1}{\Pr.Le}\phi' - \frac{K}{4}\phi = 0 \quad (11)$$

$$\frac{1}{Sc}(\eta\chi'' + \chi') - \frac{Pe}{Sc}\left(\eta(\chi'\phi' + \chi\phi'') + \frac{1}{2}\chi\phi\right) = 0 \quad (12)$$

The associted boundary conditions are

$$f(\eta_0) = 0,\ \theta(\eta_0) = 1,\ \phi(\eta_0) = 1,\ \chi(\eta_0) = 1$$
$$f(1) = b,\ \theta(1) = \delta\theta,\ \phi(1) = \delta\phi,\ \chi(1) = \delta\chi \quad (13)$$

where the prime denotes derivative with respect to $\eta$, and $\eta_0 = \left(\frac{a}{b}\right)^2$ which corresponds to the inner cylinder radius, $S = \frac{1}{b}\sqrt{\frac{\eta_1}{\mu}}$ is the couple-stress parameter , $K = \frac{k_1 b^2}{\nu}$ is the chemical reaction parameter and $Br = \frac{\mu\Omega^2}{\alpha(T_b - T_a)}$ is the Brinkman number, $\Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $Le = \frac{\alpha}{D_B}$ is the Lewis number, $Sc = \frac{\nu}{D_n}$ is the bioconvection Schmidt number, $Pe = \frac{b_c m_c}{D_n}$ is the bioconvection Peclet-number and $\delta_\theta = \frac{T_b - T_0}{T_a - T_0}$, $\delta_\phi = \frac{C_b - C_0}{C_a - C_0}$ and $\delta_\chi = \frac{\vartheta_b}{\vartheta_a}$ are constants.

The physical quantities of practical interest, which describe the flow characteristics about microorganism transfer rate is the local wall motile microorganism flux $Q_x = \frac{x}{k(\vartheta_b - \vartheta_a)}\left[\frac{\partial\vartheta}{\partial\eta}\right]_{r=a and r=b}$ are calculated. The non-dimensional form of this quantities is given by $Re^{-\frac{1}{2}}Q_x = -\chi'(\eta)$ at $\eta = \eta_0$ and $\eta = 1$.

## III. METHOD OF SOLUTION

The system of nonlinear coupled differential equations (9) - (12) are reduced to a system of linear ordinary differential equations using the successive linearization method (SLM)([14,15]).

Let, $\mathbb{F}(\eta) = [f(\eta), h(\eta), \theta(\eta), \phi(\eta), \chi(\eta)]$ and assumed that it can be expressed as

$$\mathbb{F}(\eta) = \mathbb{F}_{\tilde{\imath}}(\eta) + \sum_{k=0}^{\tilde{\imath}-1} \mathbb{F}_k(\eta), \quad \tilde{\imath} = 1,2,3,\ldots \quad (14)$$

where $\mathbb{F}_{\tilde{\imath}}(\eta)$ are unknown functions and $\mathbb{F}_k(\eta)$, $(0 \le k \le (\tilde{\imath} - 1))$ are approximations from the previous iterations. Substituting Eq.(14) in the governing equations (9) to (12) and retaining the linear terms of $\mathbb{F}_{\tilde{\imath}}(\eta)$, we obtain the following the system of ordinary differential equations.

$$a_{1,\tilde{\imath}-1} f_{\tilde{\imath}}'''' + a_{2,\tilde{\imath}-1} f_{\tilde{\imath}}''' - a_{3,\tilde{\imath}-1} f_{\tilde{\imath}}'' = r^*_{1,\tilde{\imath}-1} \quad (15)$$

$$b_{1,\tilde{\imath}-1} f_{\tilde{\imath}}'' + b_{2,\tilde{\imath}-1} f' + b_{3,\tilde{\imath}-1} f_{\tilde{\imath}} + b_{4,\tilde{\imath}-1}\theta_{\tilde{\imath}}'' + b_{5,\tilde{\imath}-1}\theta_{\tilde{\imath}}' = r^*_{2,\tilde{\imath}-1} \quad (16)$$

$$c_{1,\tilde{\imath}-1}\phi_{\tilde{\imath}}'' + c_{2,\tilde{\imath}-1}\phi_{\tilde{\imath}}' = r^*_{3,\tilde{\imath}-1} \quad (17)$$

$$d_{1,\tilde{\imath}-1}\phi_{\tilde{\imath}}'' + d_{2,\tilde{\imath}-1}\phi_r' + d_{3,\tilde{\imath}-1}\chi_{\tilde{\imath}}'' + d_{4,\tilde{\imath}-1}\chi_{\tilde{\imath}}' + d_{5,\tilde{\imath}-1}\chi_{\tilde{\imath}} = r^*_{4,\tilde{\imath}-1} \quad (18)$$

where the coefficients $a_{l,\ \tilde{\imath}-1}$, $b_{m,\ \tilde{\imath}-1}$, $c_{n,\ \tilde{\imath}-1}$, $d_{m,\ \tilde{\imath}-1}$ and $r^*_{s,\ \tilde{\imath}-1}$ where l =1, 2, 3 , m = 1, 2, 3, 4, 5, n = 1, 2, and s = 1, 2, 3,4 are in terms of the approximations $\mathbb{F}_{\tilde{\imath}}(\eta)$, $(0 \le k \le (\tilde{\imath} - 1))$ and their derivatives.

The respective boundary conditions associated with the above equations are

$f_{\tilde{\imath}-1}(\eta_0) = 0$, $\theta_{\tilde{\imath}-1}(\eta_0) = 1$, $_{\tilde{\imath}-1}(\eta_0) = 1$, $_{\tilde{\imath}-1}(\eta_0) = 1$,

$f_{\tilde{\imath}-1}(\eta = 1) = b$, $\theta_{\tilde{\imath}-1}(\eta = 1) = \delta\theta$, $\phi_{\tilde{\imath}-1}(\eta = 1) = \delta\phi$, $\chi_{\tilde{\imath}-1}(\eta = 1) = \delta\chi$ (19)

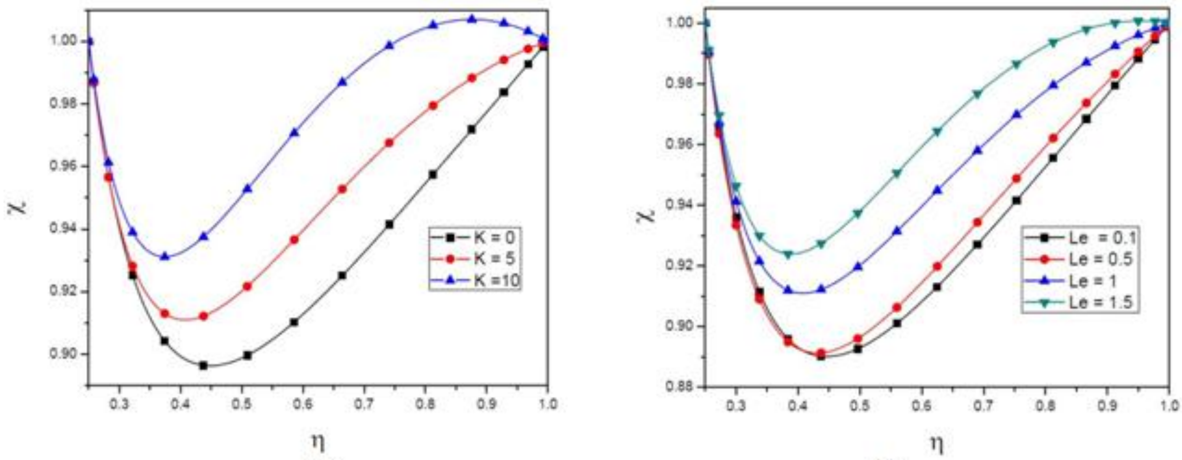
Now, we solve the linearized equations (15) to (18) by Chebyshev collocation method [16]. To implement it, frst the domain $[\eta_0, 1]$ is converted to $[-1,1]$ by the conversion $\eta = \frac{(1-\eta_0)\tau_m+(1+\eta_0)}{2}$, $-1 \leq \tau_m \leq 1$. In this procedure, the functions $\mathbb{F}_r(\eta)$ and its derivatives are approximated by the Chebyshev polynomials and these polynomials are collocated at $J+1$ Gauss-Lobatto points in the interval $[-1,1]$

Substituting these approximations into the equations (15)-(19) leads to a system of algebraic equations. Writing the boundary conditions interms of the Chebyshev polynomials at the collocation points, incorporating them in the system and solving the resulting matrix system, we get the solution.

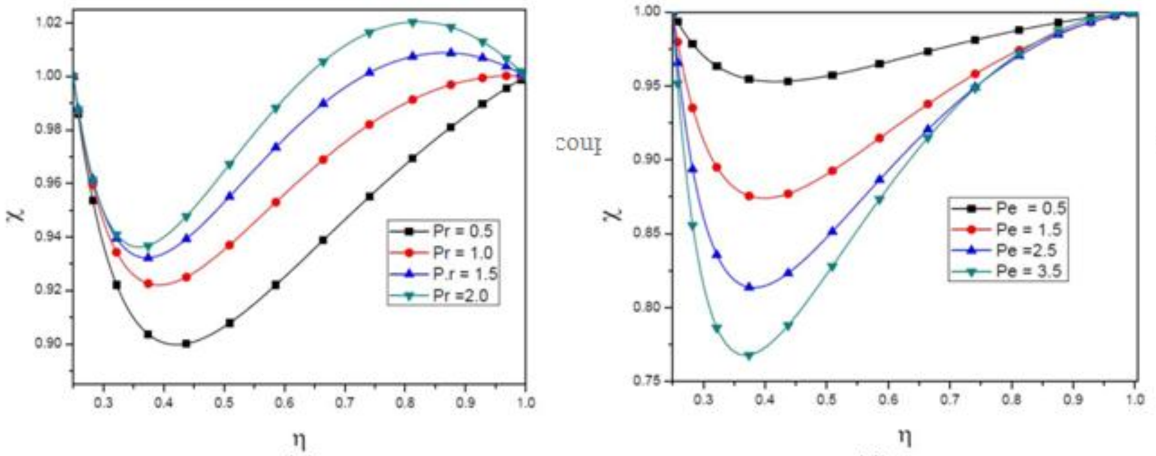
## IV. RESULTS AND DISCUSSION

The effect of various non-dimensional physical parameters such as the couple stress parameter $S$, Chemical reaction parameter $K$, the Brinkman number $Br$, the Prandtl number $Pr$, the Lewis number $Le$, the bioconvection Schmidt number $Sc$ and the bioconvection Peclet number $Pe$ on the non-dimensional density of motile microorganisms are presented graphically in Figs.2-7. An extensive computations have been carried out by assigning default values as $S$ = 0.2, $N$ = 0.5 , $Br$ = 0.1 , $K$ = 5 , $Pr$ = 0.71 , $Le$ = $Sc$ = $Pe$ = 1 and $A$ = 1 which are not against the physical requirements.

The variation of motile microorganism with various parameters such as $K$, $Le$ and $Pr$ are visualized in Fig.(2). It is perceived from Fig. (2a) that the motile microorganism increases as the chemical reaction parameter increase. For a fixed value of $K$, the motile microorganism is decreasing at the inner cylinder and increasing towards the rotating outer cylinder via concave upward pattern. The reason can be understood that as the outer cylinder rotates the motile microorganisms are assumed to gather at the outer cylinder which causes an increase in the density of the motile microorganism. From Figs.(2b)-(2c)), it is depicted that the motile microorganisms is rising with the rise in the values of $Le$ and $Pr$. The variation of $\chi$ for varying bioconvection parameter $Pe$ is plotted in Fig.(2d). For a fixed $Pe$, the motile microorganism is reducing initially near the inner cylinder till the mid of the annulus region and then rising towards the outer cylinder. Further, it is noticed that $Pe$ increases the density of motile microorganism decreases.

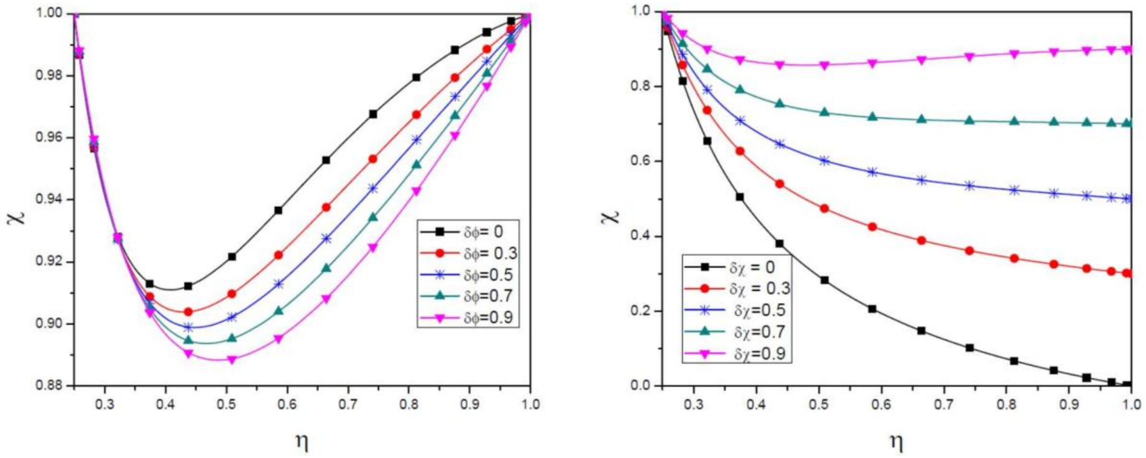



(a) (b)




(c) (d)

**Figure 2:** Profiles of $\chi$ with the parameters (a) chemical reaction parameter ($K$), (b) Lewis Number ($Le$), (c) Prandtl number (Pr) and (d) Peclet number $Pe$

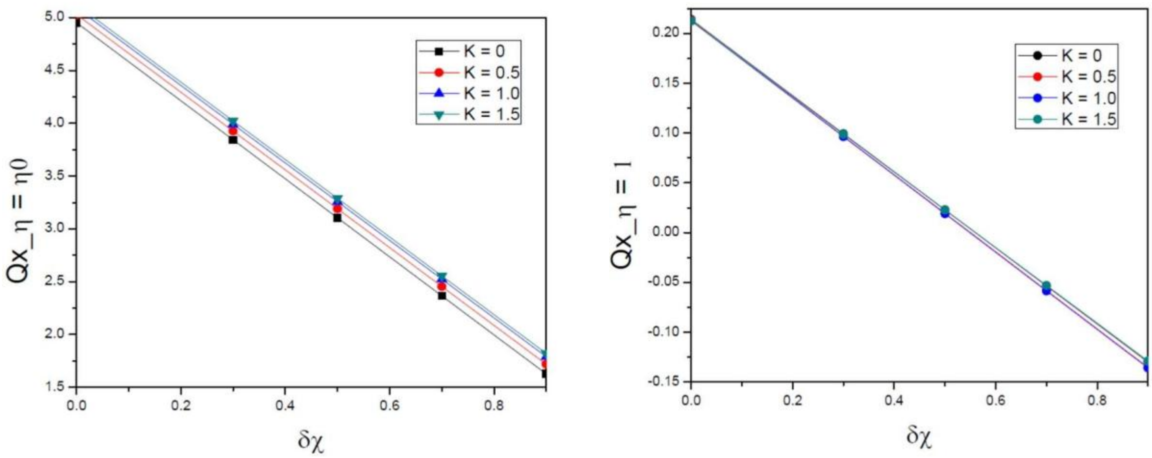



(a) (b)

**Figure 3:** Profiles of $\chi$ with the slip parameters (a) $\delta\phi$ and (b) $\delta\chi$

The influence of concentration slip parameter $\delta\phi$ and the motile microorganism slip parameter $\delta\chi$ on the motile microorganisms is exhibited in the Figs.(3). The Motile microorganisms, as illustrated in Fig.(3a), is inversely proportional to $\delta\phi$. For a given $\delta\phi$ a notable impact on the behaviour of motile microorganisms $\chi(\eta)$ is observed i.e., the density of motile microorganism is reducing near the inner cylinder and increasing towards the outer cylinder. As the $\delta\chi$ increases the motile microorganism is also increasing, as depicted in Fig. (3b), It is clear that the density motile microorganism decreases for a given value of the slip parameter value $\delta\chi$ as shown Fig. (3b).

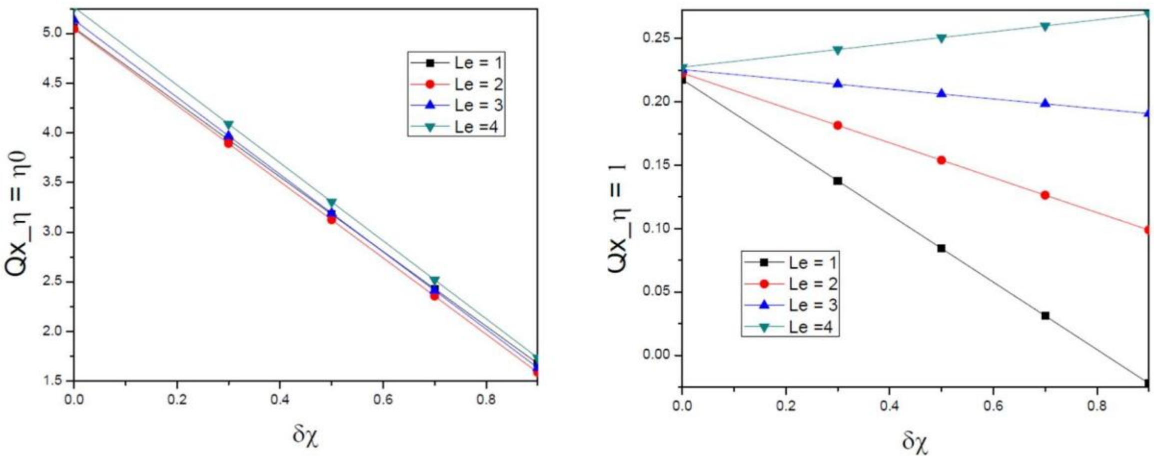
The variation of the density number of motile microorganism ($Q_{\mathrm{X}}$ ) with the parameters $K$ with respect to $\delta\chi$ is presented in Fig. 4. The profile of $Q_{\mathrm{X}}$ at the inner cylinder and outer cylinders is exhibited Fig. 4(a) and Fig. 4(b). It is noticed from Fig. 4 that the density number at the inner and outer cylinders is decreasing with an increase in $\delta\chi$. Further, it is observed that $Q_{\mathrm{X}}$ is directly proportional to K as shown in Fig. 4(a). Similarly, it is seen from Fig. 4(b) that the influence of K on $Q_{\mathrm{X}}$ at the outer cylinder is negligible.




(a) (b)

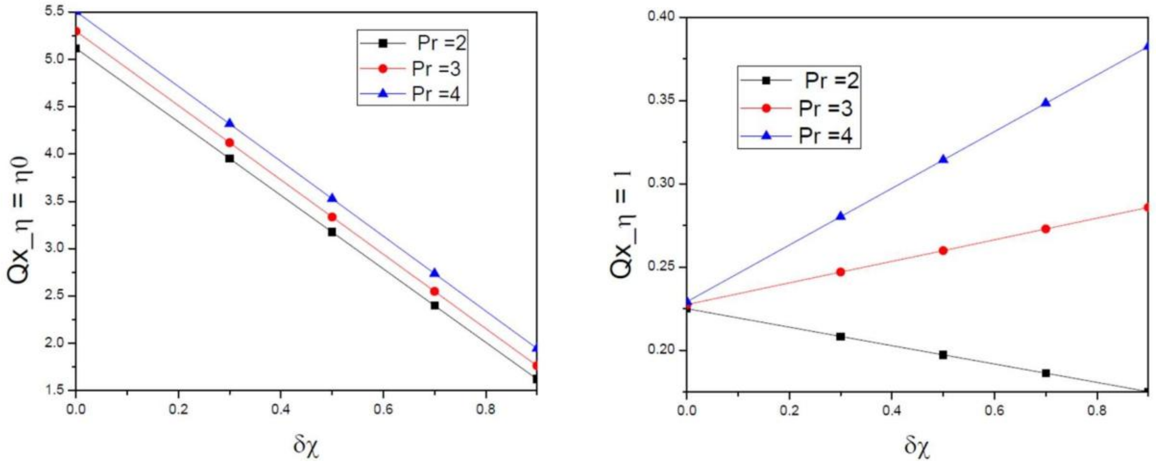
Figure 4 : Variation of $Q_{\mathrm{X}}$ with $\delta\chi$ for different values of $K$ at the boundary of (a) inner cylinder and (b) outer cylinders

The influence of Lewis number on the density number of motile microorganism ($Q_{\mathrm{X}}$) with $\delta\chi$ is depicted in Fig. 5. It is noticed from Fig. 5(a) that the density number at the inner is increasing with an increase in the parameter Le. As the lewis number is increasing, $Q_{\mathrm{X}}$ at the outer cylinder is also increasing as depicted in Fig. 5(b). The effect of Prandtl number on $Q_{\mathrm{X}}$ with $\delta\chi$ is depicted in Fig. 6. It is noticed from Fig. 6(a) that the density number at the inner is increasing with an increase in Pr. As the Prandtl number is increasing, $Q_{\mathrm{X}}$ at the outer cylinder, is also increasing as depicted in Fig. 6(b).




(a) (b)

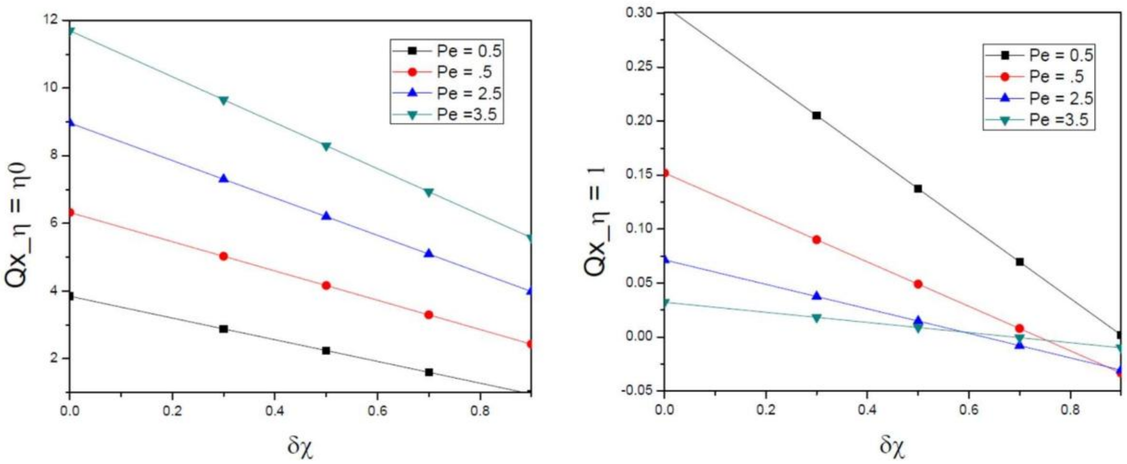
Figure 5 : Variation of $Q_{\mathrm{X}}$ with $\delta\chi$ for different values of $Le$ at the boundary of (a) inner cylinder and (b) outer cylinders




(a) (b)

Figure 6 : Variation of $Q_{\mathrm{X}}$ with $\delta\chi$ for different values of $Pr$ at the boundary of (a) inner cylinder and (b) outer cylinders

Figure 7 shows the effect of Peclet number on $Q_{\mathrm{X}}$ with $\delta\chi$. It is noticed from Fig. 7(a) that the density number at the inner is increasing with an increase in Pe. As the Peclet-number is increasing $Q_{\mathrm{X}}$ at the outer cylinder is decreasing as depicted in Fig. 7(b).




(a) (b)

Figure 7 : Variation of $Q_{\mathrm{X}}$ with $\delta\chi$ for different values of $Pe$ at the boundary of (a) inner Cylinder and (b) outer cylinders

## V. CONCLUSIONS

The present study gives numerical solutions for the flow of a incompressible couple stress fluid containing microorganisms between two concentric cylinders with the rotating outer cylinder.

☐ The motile microorganism $\chi$ increases as $K$, $Le$, $Pr$ increases and it decreases as $Pe$ number increase. Furhter, $\chi$ is indreictly proportionl to $\delta\phi$ and directly proportional $\delta\chi$.

☐ The motile microorganism is reducing initially near the inner cylinder till the mid of the annulus region and then rising towards the outer cylinder.

☐ At the inner cylinder, the $Q_{\mathrm{X}}$ is increasing function of $K$, $Le$, $Pr$ and $Pe$. At the outer cylinder, $Q_{\mathrm{X}}$ is decreasing as $K$ and $Pe$ increase. As the values of $Le$ and $Pr$ are increasing $Q_{\mathrm{X}}$ is also increasing at the outer cyliner.

☐ For a given value of $K$, $Le$, $Pr$ and $Pe$, $Q_{\mathrm{X}}$ shows a decreasing pattern with an increase in $\delta\chi$ at the inner cylinder. At the outer cylinder, the $Q_{\mathrm{X}}$ is decreasing with an increase in $\delta\chi$ for a given value of $K$, $Le$, and $Pe$ and reverse trend for fixed value of $Pr$.

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